

A Numerical Study of the Performance of a Quantum Adiabatic Evolution Algorithm for Satisfiability

Edward Farhi, Jeffrey Goldstone

Center for Theoretical Physics
Massachusetts Institute of Technology
Cambridge, MA 02139
farhi@mit.edu goldstone@mitlms.mit.edu

Sam Gutmann

Department of Mathematics
Northeastern University
Boston, MA 02115
sgutm@neu.edu

MIT-CTP #3006

Abstract

Quantum computation by adiabatic evolution, as described in quant-ph/0001106, will solve satisfiability problems if the running time is long enough. In certain special cases (that are classically easy) we know that the quantum algorithm requires a running time that grows as a polynomial in the number of bits. In this paper we present numerical results on randomly generated instances of an NP-complete problem and of a problem that can be solved classically in polynomial time. We simulate a quantum computer (of up to 16 qubits) by integrating the Schrödinger equation on a conventional computer. For both problems considered, for the set of instances studied, the required running time appears to grow slowly as a function of the number of bits.

1 Introduction

A quantum algorithm for the satisfiability problem was presented in [1]. This algorithm is based on quantum adiabatic evolution. If a state $|\psi(t)\rangle$ evolves according to the Schrödinger equation with a slowly varying Hamiltonian $H(t)$ and $|\psi(0)\rangle$ is the ground state of $H(0)$, then $|\psi(t)\rangle$ will stay close to the instantaneous ground state of $H(t)$. The Hamiltonian $H(t)$ used in the algorithm is designed so that the ground state of $H(0)$ is easy to construct and the ground state of $H(T)$ encodes the solution to the instance of satisfiability. A crucial question is how large must the running time, T , be to achieve an acceptable probability of success. In this paper we simulate an n -qubit continuous time quantum computer by

numerically integrating the Schrödinger equation in a 2^n -dimensional Hilbert space. We randomly generate difficult instances of an NP-complete problem and study how large T must be as a function of the number of bits n . For $7 \leq n \leq 16$, we find that the required T grows modestly with n since the data is well fit by a quadratic in n .

2 A quantum algorithm based on adiabatic evolution

An n -bit instance of satisfiability is a Boolean formula with M clauses

$$C_1 \wedge C_2 \wedge \cdots \wedge C_M \quad (2.1)$$

where each clause C_a is True or False depending on the values of some subset of the n bits. The task is to discover if one (or more) of the 2^n possible assignments of the values of the n bits satisfies all of the clauses, that is, makes formula (2.1) True. We consider two restricted versions of satisfiability, both of which are restricted versions of a problem called “exact cover”. In the first, EC3, each clause involves only three bits. The clause always has the same form. The clause is True if and only if one of the three bits is a 1 and the other two are 0, so there are three satisfying assignments out of the eight possible values of the three bits. The second, EC2, has the restriction that each clause involves only two bits. In this case the clause is True if and only if the two bits have the value 01 or 10, so there are two satisfying assignments out of the four possible values for the two bits. We pick these two examples because EC2 is classically solvable in polynomial time whereas EC3 is NP-complete and no polynomial time algorithm is known. It is interesting to see how the quantum algorithm treats these two classically very different problems.

To understand the quantum algorithm we first recall the content of the adiabatic theorem. We are given a Hamiltonian $\tilde{H}(s)$ that depends smoothly on the parameter s where s varies from 0 to 1. Suppose that for each value of s , $\tilde{H}(s)$ has a unique lowest energy state, the ground state $|g; s\rangle$. That is,

$$\tilde{H}(s)|g; s\rangle = E_0(s)|g; s\rangle \quad (2.2)$$

where $E_0(s)$ is strictly less than any of the other eigenvalues of $\tilde{H}(s)$. Introduce a time scale T and define a time-dependent Hamiltonian $H(t)$ by

$$H(t) = \tilde{H}(t/T) \quad (2.3)$$

where t varies from 0 to T . As T gets bigger, $H(t)$ becomes more slowly varying. Let $|\psi(t)\rangle$ obey the Schrödinger equation

$$i \frac{d}{dt} |\psi(t)\rangle = H(t) |\psi(t)\rangle \quad (2.4)$$

with

$$|\psi(0)\rangle = |g; s = 0\rangle. \quad (2.5)$$

That is, at $t = 0$, $|\psi(0)\rangle$ is the ground state of $H(0)$. The adiabatic theorem tells us that

$$\lim_{T \rightarrow \infty} |\langle g; s = 1 | \psi(T) \rangle| = 1. \quad (2.6)$$

This means that for T large enough, $|\psi(T)\rangle$ is (up to a phase) very close to the ground state of $H(T)$. Eq. (2.6) only holds if the gap between the ground state energy, $E_0(s)$ of $\tilde{H}(s)$, and the next highest energy, $E_1(s)$ of $\tilde{H}(s)$, is strictly greater than zero, that is, $E_1(s) - E_0(s) > 0$ for $0 \leq s \leq 1$. We will also discuss cases where at $s = 1$ there are multiple ground states. In this situation the evolution ends very close to the ground state subspace.

The idea behind the quantum algorithm is the following. Given an instance of the satisfiability problem it is straightforward to construct a Hamiltonian, H_P , whose ground state corresponds to an assignment of the bits that violates the least number of clauses. Although it is easy to construct H_P , finding its ground state may be computationally challenging. However we can construct a Hamiltonian H_B whose ground state we explicitly know. Now let

$$\tilde{H}(s) = (1 - s)H_B + sH_P \quad (2.7)$$

and accordingly

$$H(t) = (1 - t/T)H_B + (t/T)H_P \quad (2.8)$$

for some fixed T . We start our quantum system in the known ground state of H_B and then evolve according to Eq. (2.4) for time T . Suppose that the instance of satisfiability that gave rise to H_P has a unique satisfying assignment. If T is large enough, then $|\psi(T)\rangle$ will be very close to the ground state of H_P . Measuring $|\psi(T)\rangle$ will, with high probability, produce the satisfying assignment. In general this algorithm will produce an assignment that violates the minimum number of clauses in Eq. (2.1). Restricting to instances with a unique satisfying assignment appears to pick out difficult instances and simplifies the analysis of our algorithm.

More explicitly, given an n -bit instance of satisfiability, we work in an n -qubit Hilbert space that is spanned by the 2^n basis vectors $|z_1\rangle|z_2\rangle \cdots |z_n\rangle$ where $z_i = 0, 1$ and $|z_i\rangle$ is an eigenstate of the z component of the i^{th} spin,

$$\frac{1}{2}(1 - \sigma_z^{(i)})|z_i\rangle = z_i|z_i\rangle \quad \text{where each} \quad \sigma_z^{(i)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (2.9)$$

We also need the eigenstates of the x component of the i^{th} spin,

$$|x_i = 0\rangle = \frac{1}{\sqrt{2}}(|z_i = 0\rangle + |z_i = 1\rangle)$$

and

$$|x_i = 1\rangle = \frac{1}{\sqrt{2}}(|z_i = 0\rangle - |z_i = 1\rangle) \quad (2.10)$$

that obey

$$\frac{1}{2}(1 - \sigma_x^{(i)})|x_i = x\rangle = x|x_i = x\rangle \quad \text{where each} \quad \sigma_x^{(i)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (2.11)$$

For concreteness imagine that each clause in formula (2.1) involves 3 bits. Let i_C, j_C and k_C be the 3 bits associated with clause C . For each clause C define an “energy” function

$$h_C(z_{i_C}, z_{j_C}, z_{k_C}) = \begin{cases} 0, & \text{if } (z_{i_C}, z_{j_C}, z_{k_C}) \text{ satisfies clause } C \\ 1, & \text{if } (z_{i_C}, z_{j_C}, z_{k_C}) \text{ violates clause } C. \end{cases} \quad (2.12)$$

We immediately turn these into quantum operators

$$H_{P,C}(|z_1\rangle|z_2\rangle \cdots |z_n\rangle) = h_C(z_{i_C}, z_{j_C}, z_{k_C})|z_1\rangle|z_2\rangle \cdots |z_n\rangle \quad (2.13)$$

and define

$$H_P = \sum_C H_{P,C}. \quad (2.14)$$

By construction H_P is nonnegative, that is, $\langle \psi | H_P | \psi \rangle \geq 0$ for all $|\psi\rangle$ and $H_P|\psi\rangle = 0$ if and only if $|\psi\rangle$ is a superposition of states of the form $|z_1\rangle|z_2\rangle \cdots |z_n\rangle$ where the bit string $z_1 z_2 \dots z_n$ satisfies all of the clauses. In this context seeing if formula (2.1) has a satisfying assignment is accomplished by finding the ground state of H_P . If formula (2.1) has no satisfying assignment, the ground state (or states) of H_P correspond to the assignment (or assignments) that violate the least number of clauses.

H_P given by Eq. (2.14) is the problem Hamiltonian whose ground state we seek when we run the quantum algorithm. For the beginning Hamiltonian H_B , first define

$$H_B^{(i)} = \frac{1}{2}(1 - \sigma_x^{(i)}). \quad (2.15)$$

For each clause C , involving bits i_C, j_C , and k_C , let

$$H_{B,C} = H_B^{(i_C)} + H_B^{(j_C)} + H_B^{(k_C)}. \quad (2.16)$$

The beginning Hamiltonian is given by

$$H_B = \sum_C H_{B,C}. \quad (2.17)$$

The ground state of H_B is

$$|x_1 = 0\rangle|x_2 = 0\rangle \cdots |x_n = 0\rangle = \frac{1}{2^{n/2}} \sum_{z_1} \sum_{z_2} \cdots \sum_{z_n} |z_1\rangle|z_2\rangle \cdots |z_n\rangle \quad (2.18)$$

which we will take to be the initial state when we run the quantum algorithm.

The Hamiltonian that governs the evolution of the quantum system is given by Eq. (2.8) with H_P specified in Eq. (2.14) and H_B specified in Eq. (2.17). Note that $H(t)$ is a sum of terms, each of which is associated with one of the clauses in (2.1),

$$H(t) = \sum_C H_C(t) \quad (2.19)$$

where

$$H_C(t) = (1 - t/T)H_{B,C} + (t/T)H_{P,C}. \quad (2.20)$$

Each $H_C(t)$ involves only the bits associated with clause C and therefore $H(t)$ is a sum of terms each of which involves a fixed number of bits.

For a given problem such as EC3 or EC2 we must specify the running time as a function of the number of bits, $T(n)$. Since the state $|\psi(T)\rangle$ is not exactly the ground state of H_P we must also specify $R(n)$, the number of times we repeat the quantum evolution in order to have a desired probability of success. This paper can be viewed as an attempt to determine $T(n)$ and $R(n)$ by numerical methods.

We now summarize the ingredients of the quantum adiabatic evolution algorithm. Given a problem and an n -bit instance, we assume we know the instance-independent running time $T(n)$ and repetition number $R(n)$. For the instance and given $T(n)$,

1. Construct the time-dependent Hamiltonian $H(t)$ given by Eq. (2.8) with H_B and H_P given by Eq. (2.14) and Eq. (2.17).
2. Start the quantum system in the state $|\psi(0)\rangle$ given by Eq. (2.18).
3. Evolve according to Eq. (2.4) for a time $T = T(n)$ to arrive at $|\psi(T)\rangle$.
4. Measure z_1, z_2, \dots, z_n in the state $|\psi(T)\rangle$ and check if the bit string $z_1 z_2 \dots z_n$ satisfies all clauses.
5. Repeat $R(n)$ times.

3 Choosing instances

For our numerical study we randomly generate instances of the two problems under study, EC3 and EC2. Focus first on EC3. For a decision problem, it suffices to produce a satisfying assignment when one or more exists. For now, we restrict our attention to instances with a unique satisfying assignment. We believe that instances with only one satisfying assignment include most of the difficult instances for our algorithm. In fact as we will see later, our algorithm runs faster on instances with more than one satisfying assignment so the restriction to a unique satisfying assignment appears to restrict us to the most difficult cases.

With the number of bits fixed to be n , we generate instances of EC3 as follows. We pick three bits at random, uniformly over the integers from 1 to n . (The bits must all be different.) We then have a formula with one exact cover clause. We calculate the number of satisfying assignments. We then add another clause by picking a new set of three bits. Again we calculate the number of satisfying assignments. We continue adding clauses until the number of satisfying assignments is reduced to one or zero. If there is one satisfying assignment we accept the instance. If there are none we reject the instance and start again. Using this procedure the number of clauses is not a fixed function of the number of bits but rather varies from instance to instance. For EC3 we find that the number of clauses is typically close to the number of bits.

We follow a similar procedure for EC2. When we add a clause we randomly specify which two bits are involved in the clause. Again we repeat this procedure until there are two satisfying assignments (or none in which case we discard the instance). We stop at two satisfying assignments because EC2 has a bit negation symmetry. If $w_1 w_2 \dots w_n$ is a satisfying assignment so is $\bar{w}_1 \bar{w}_2 \dots \bar{w}_n$ and accordingly there are no instances with a single satisfying assignment. For EC2 the number of clauses is typically close to the number of bits.

We know that EC2 is classically computationally simple but of course there is no guarantee that quantum adiabatic evolution will work well on EC2. We choose instances of EC2 with two satisfying assignments to parallel as closely as possible our study of EC3.

4 Numerical simulation

We are exploring the quantum adiabatic evolution algorithm by numerically simulating a perfectly functioning quantum computer. The quantum computer takes an initial state $|\psi(0)\rangle$, given by Eq. (2.18), and evolves it according to the Schrödinger equation (2.4), for a time T , to produce $|\psi(T)\rangle$. The Hamiltonian $H(t)$ is given by Eq. (2.8) with H_P and H_B determined by the instances of satisfiability being studied.

If the number of bits is n , the dimension of the Hilbert space is 2^n . This exponential growth in required space is the well-known impediment to simulating a quantum computer with many bits. For our modest numerical investigation, using Macs running MATLAB for a few hundred hours, we can only explore out to 16 bits. We integrate the Schrödinger equation using a variable step size Runge-Kutta algorithm, checking accuracy by keeping the norm of the state equal to unity to one part in a thousand. Since the number of bits is modest, we can always explicitly determine the ground state (or ground states) of H_P . Given $|\psi(T)\rangle$ and the ground state (or states) of H_P we can calculate the probability that a measurement of z_1, z_2, \dots, z_n will give a satisfying assignment by taking the sum of the squares of the inner products of $|\psi(T)\rangle$ with $|w_1\rangle|w_2\rangle \dots |w_n\rangle$ where the bit strings $w_1 w_2 \dots w_n$ are the satisfying assignments.

5 The median time to achieve probability 1/8

Our goal in this paper is to explore the running time $T(n)$ and the repetition number $R(n)$ that will give a successful algorithm. To this end we first determine the typical running time needed to achieve a fixed probability of success for a randomly generated instance with n bits for $7 \leq n \leq 15$. In particular we determine the median time required to achieve a success probability of 1/8. Since this is a numerical study we actually hunt for a time that produces a probability between 0.12 and 0.13. For each n between 7 and 15, for both EC3 and EC2, we find the median of 50 randomly generated instances.

In Figure 1 the circles represent the data for EC3 and the solid curve is a quadratic fit

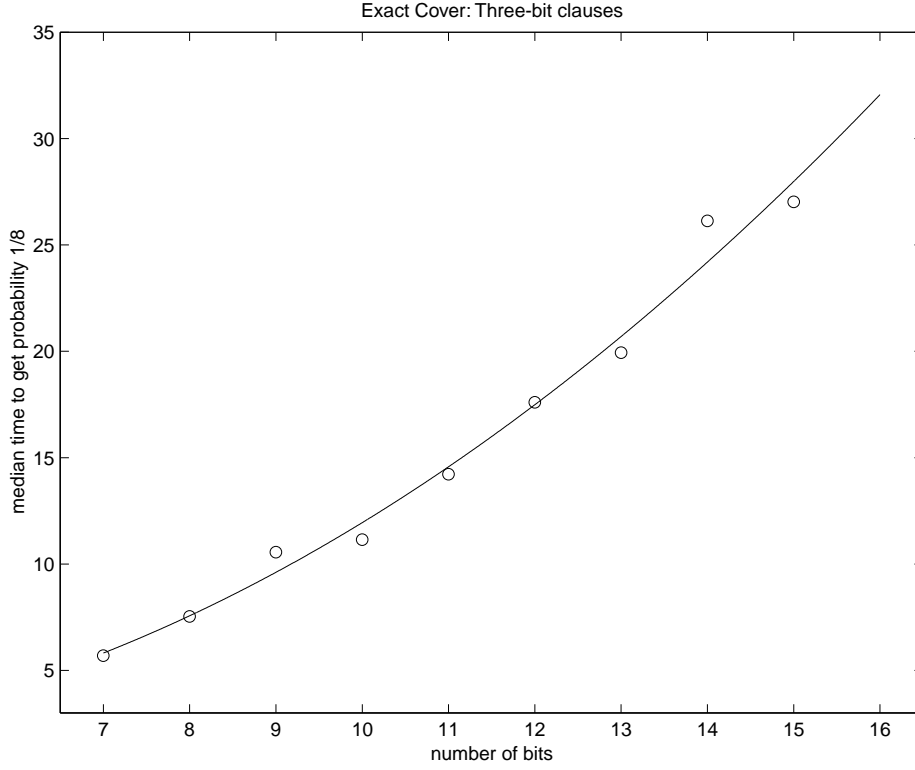


Figure 1: Exact Cover with three-bit clauses. Each circle is the median time to achieve probability between 0.12 and 0.13 for 50 instances. The curve is a quadratic fit to the data.

to the data. At each number of bits the times required to reach probability $1/8$ range from roughly half the median to twice the median. For this range of n , a quadratic, or even linear fit, is clearly adequate. The exponential $1.618 (1.215)^n$ is also a good fit. In the next section we show a situation where an exponential fit to the data is *required* for the same range of n .

We know of one anomalous instance (discovered by Daniel Preda, outside of the data collected for this paper) with 11 bits and a time to achieve probability $1/8$ of close to 300. However, at $T = 14.57$, which is the value of the quadratic fit in Figure 1 at $n = 11$, the probability of success for this instance is already 0.0606. Because this probability is not anomalously low, the algorithm proposed in Section 7 will have no difficulty with this instance.

In Figure 2 the circles represent the data for EC2 and the solid curve is a linear fit to the data. Here the maximum time required to reach probability $1/8$, for each number of bits, is roughly 20% greater than the median.

6 Destroying the bit structure of the Hamiltonian

The Hamiltonian $H(t)$ is a sum of terms $H_C(t)$ each of which involves only the few bits mentioned in clause C ; see Eq. (2.19). Each $H_C(t)$ is associated with a subspace of the

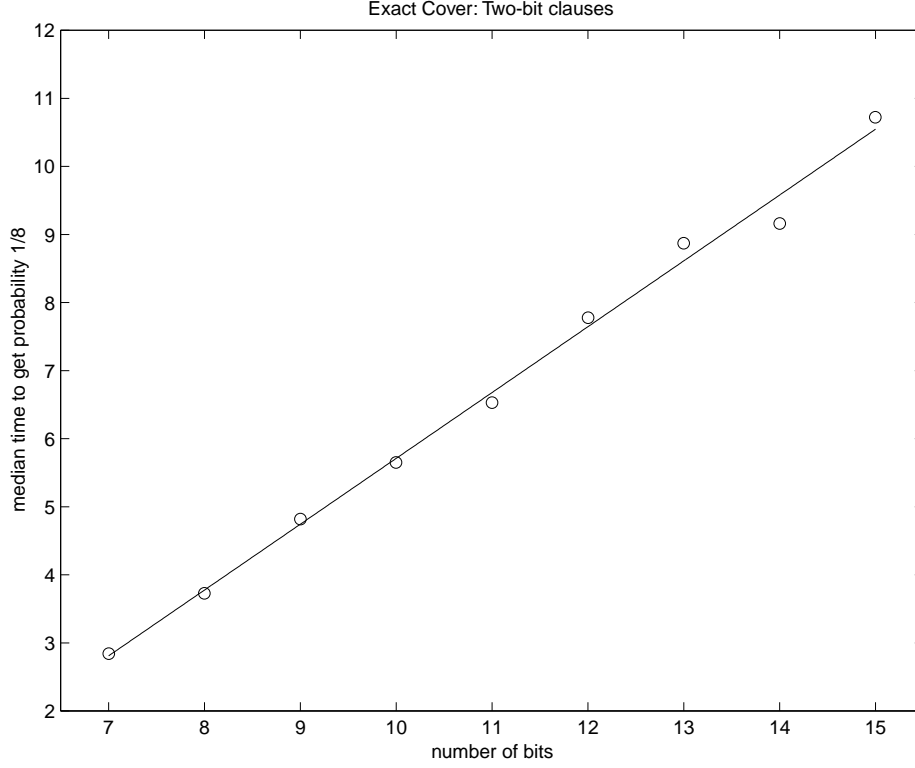


Figure 2: Exact Cover with two-bit clauses. Each circle is the median time to achieve probability between 0.12 and 0.13 for 50 instances. The curve is a linear fit to the data.

Hilbert space corresponding to the satisfying assignments of clause C , that is, the space spanned by the ground states of $H_{P,C}$; see Eq. (2.13). Quantum adiabatic evolution (for T big enough) yields a state in the intersection of the subspaces associated with all of the clauses. Our intuition is that the bit structure of these subspaces is crucial to the success of the quantum algorithm.

To test this intuition we destroy the bit structure of the Hamiltonian and run the algorithm again. Specifically consider the classical energy function that counts the number of violated clauses

$$h(z) = \sum_C h_C(z_{i_C}, z_{j_C}, z_{k_C}) \quad (6.1)$$

where $z = z_1 z_2 \dots z_n$ and h_C is given in Eq. (2.12). From Eq. (2.13) and Eq. (2.14) we have that

$$H_P(|z_1\rangle|z_2\rangle \dots |z_n\rangle) = h(z)|z_1\rangle|z_2\rangle \dots |z_n\rangle. \quad (6.2)$$

Now let

$$h_{\text{SCRAM}}(z) = h(\Pi(z)) \quad (6.3)$$

where Π is a random permutation of the integers $\{0, 1, 2, \dots, 2^n - 1\}$. Note that Π is *not* a permutation of the bits but rather a random scrambling of all 2^n of the z 's.

Let

$$H_{\text{SCRAM},P}(|z_1\rangle|z_2\rangle\cdots|z_n\rangle) = h_{\text{SCRAM}}(z)|z_1\rangle|z_2\rangle\cdots|z_n\rangle \quad (6.4)$$

and accordingly

$$H_{\text{SCRAM}}(t) = (1 - t/T) H_B + (t/T) H_{\text{SCRAM},P}. \quad (6.5)$$

The spectrum of $H_{\text{SCRAM},P}$ is identical to that of H_P but the relationship between the eigenvalues and the values of z has been scrambled. Accordingly the spectrum of $H_{\text{SCRAM}}(t)$ is not equal to the spectrum of $H(t)$ except at $t = 0$ and $t = T$. If H_P has a unique ground state so does $H_{\text{SCRAM},P}$ and for T large enough we expect (again by the adiabatic theorem) that quantum evolution by $H_{\text{SCRAM}}(t)$ will bring us to the ground state of $H_{\text{SCRAM},P}$.

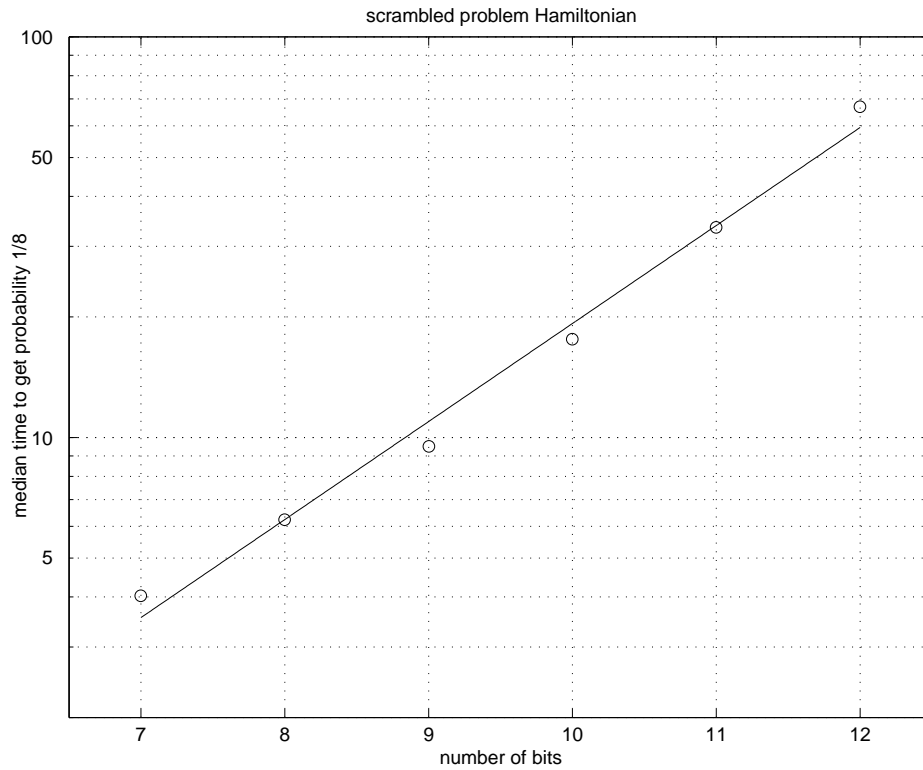


Figure 3: Exact Cover with two-bit clauses and a scrambled problem Hamiltonian. Each circle is the median time to achieve probability between 0.12 and 0.13 for 100 instances. The solid line on this log plot represents an exponential fit to the data.

Once the bit structure has been destroyed, finding the minimum of $h_{\text{SCRAM}}(z)$ is essentially the problem of minimizing an arbitrary function defined on $\{0, 1, 2, \dots, 2^n - 1\}$. Solving this problem requires exponential time even on a quantum computer[2, 3]. To confirm this we generated 100 instances of EC2 for each of $n = 7, 8, 9, 10, 11, 12$. For each instance we generate a random permutation Π and quantum evolve with $H_{\text{SCRAM}}(t)$ for time T . In Figure 3 we show the median time T required to achieve a success probability of $1/8$. The linear behavior of the data on the log plot indicates an exponential growth as a function of n and

the solid line represents $0.0689(1.7565)^n$. In contrast with the data in Figures 1 and 2, the data in Figure 3 cannot be well fit by a quadratic.

7 Probabilities of success at a proposed running time

Figures 1 and 2, at least by comparison with Figure 3, indicate that the median times required to achieve probability $1/8$, for EC3 and EC2 with $7 \leq n \leq 15$, grow modestly with n . Thus the fitted medians, for EC3 and EC2, are reasonable candidates for the running times $T(n)$ of our algorithm for these two problems. Automatically, with these run times, our algorithm will achieve a success probability of at least $1/8$ for about half of all instances. Our goal now is to explore how low the success probability can get at these run times. To this end we generate 100 new instances of EC3 and EC2 for each n . Now n runs from 7 to 16 for EC3 and from 7 to 15 for EC2 and $T(n)$ is given by the fit to the data shown in Figures 1 and 2.

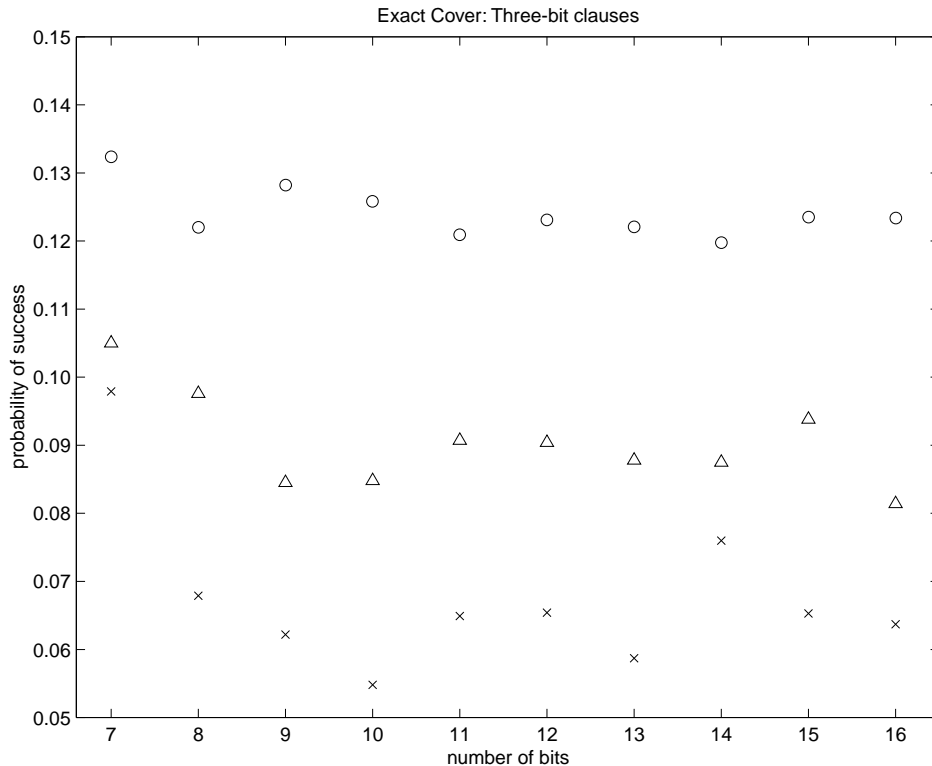


Figure 4: Exact Cover with three-bit clauses. Probability of success at the proposed running time. Circles represent the medians of 100 instances at each number of bits. Triangles are the 10th lowest probabilities and X's are the lowest.

Figure 4 displays the results for EC3. For each value of n from 7 to 16 we show the median probability of success at $T(n)$ as well as the smallest of the 100 probabilities and the 10th smallest. It is no surprise that the median probability is close to $1/8$ for all values of n .

The fact that the smallest probability does not decrease with n was not anticipated. This means that, at least for the range of the number of bits considered, the number of repetitions $R(n)$ can be taken to be constant with n to achieve a fixed desired probability of success.

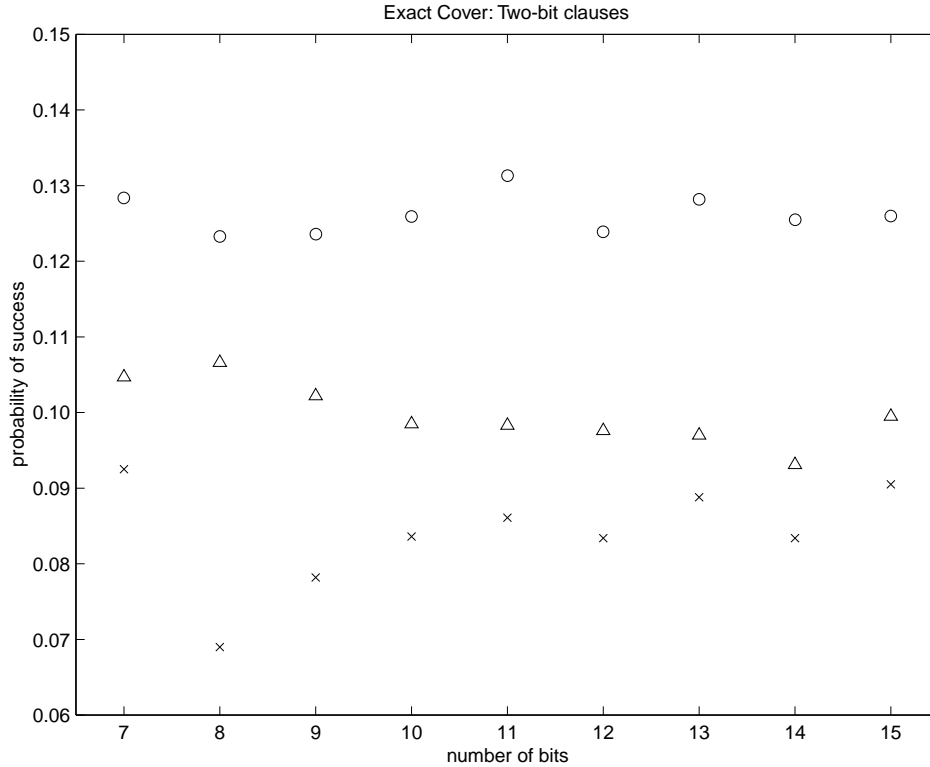


Figure 5: Exact Cover with two-bit clauses. Probability of success at the proposed running time. Circles represent the medians of 100 instances at each number of bits. Triangles are the 10th lowest probabilities and X's are the lowest.

In Figure 5 the data for EC2 is presented. Here the run time $T(n)$ is given by the linear fit to the data in Figure 2. Again the median probability at each value of n is close to $1/8$ as is expected. However, even for this classically easy problem we had no guarantee that the worst case probability would not decrease with n . In fact it does not appear to decrease at all.

In order to show that the running time $T(n)$ must increase with n to produce a successful algorithm, we explore the success probabilities obtained when using an n -independent running time. More specifically for the EC3 instances used to generate Figure 4 we run the algorithm for $n = 7, 8, \dots, 14$ for a constant value of T , the one previously used for $n = 7$. In Figure 6 the log plot shows that the median success probability decreases exponentially, which means that the number of repetitions $R(n)$ would have to grow exponentially to achieve a fixed probability of success.

In Section 6 we gave evidence that the bit structure is crucial to the success of the quantum adiabatic evolution algorithm. We make this point again by taking the instances of

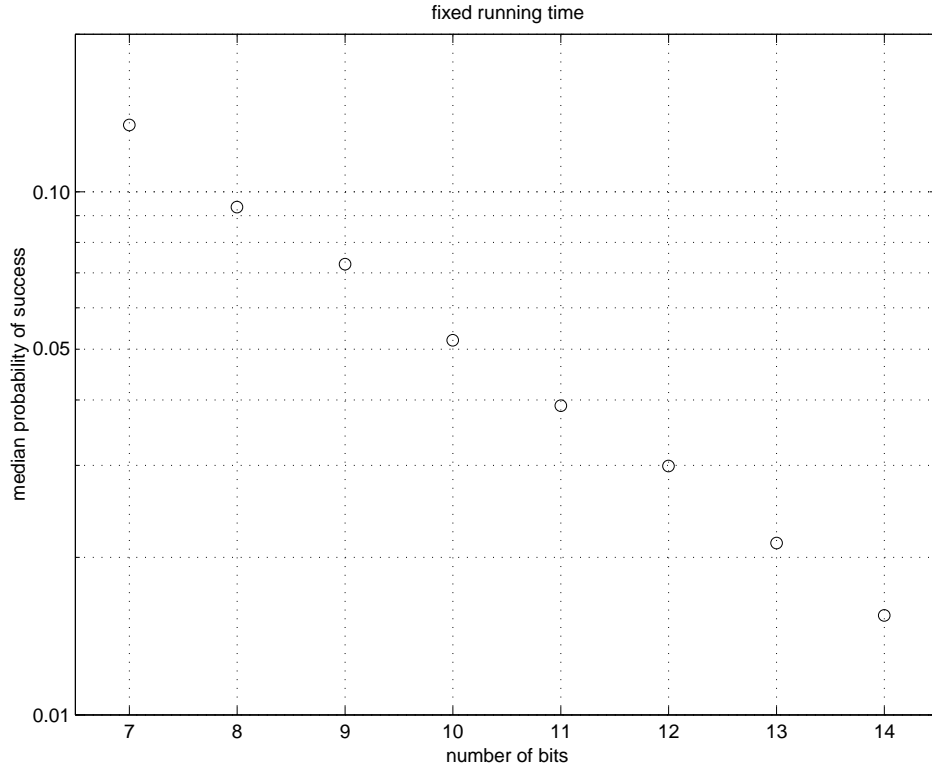


Figure 6: Exact Cover with three-bit clauses running at $T = 5.82$ for all numbers of bits. Circles represent the median probability of 100 instances. The straight line behavior on the log plot shows an exponential decrease in success probability.

EC3, for $n = 7, 8, \dots, 14$, and running the algorithm with $T(n)$ given by the fit in Figure 1 but with $H(t)$ replaced by $H_{\text{SCRAM}}(t)$. In Figure 7, the median probability of success is seen to decrease exponentially with n . This helps confirm our intuition that the quantum adiabatic evolution algorithm takes advantage of the bit structure inherent in the problem.

8 Instances of EC3 with more than one satisfying assignment

All of the EC3 data presented up to this point was generated from instances with unique satisfying assignments. Now we explore EC3 instances with more than one satisfying assignment. As in Section 3 clauses are added at random but now instances are accepted as soon as the number of satisfying assignments is 6, 7, 8, or 9. If adding a clause reduces the number of satisfying assignments from more than 9 to less than 6, the instance is rejected. We do this with 100 instances for 10, 11, 12, and 13 bits and run at the same times $T(n)$ used for instances with a unique satisfying assignment to generate Figure 4.

In Figure 8 we show the median probability, the smallest probability, and the 10th smallest for these instances. At the running times used, the median probability for instances with unique satisfying assignments is close to $1/8$ (for any number of bits). For the instances

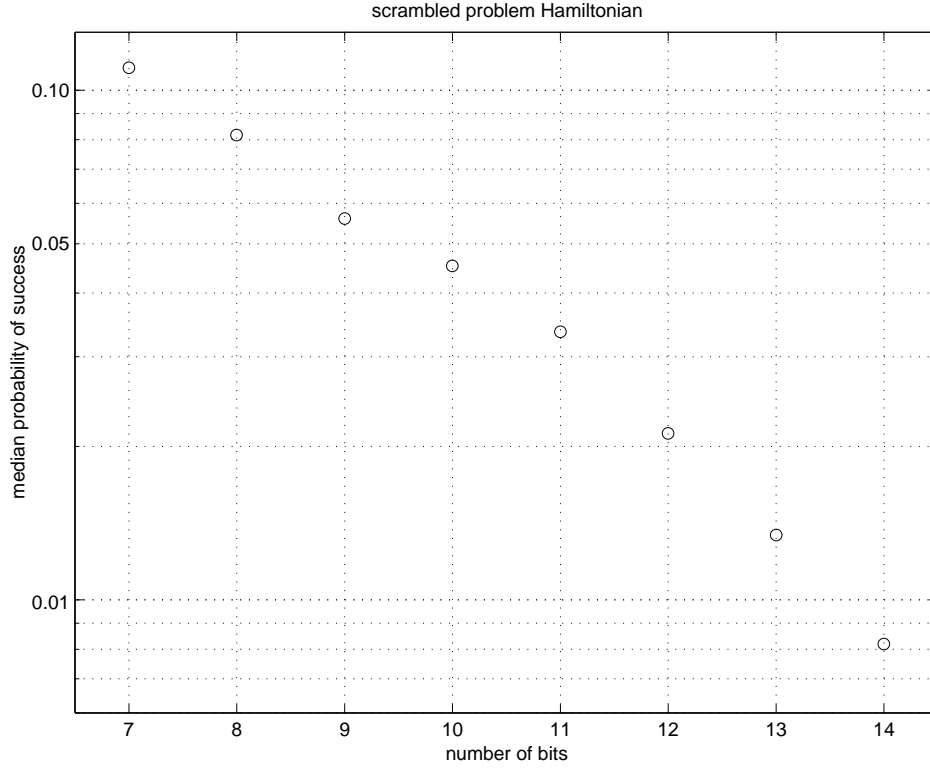


Figure 7: Exact Cover with three-bit clauses with a scrambled problem Hamiltonian. The running time is given by the quadratic fit shown in Figure 1. The log plot shows an exponential decrease in success probability.

with multiple satisfying assignments the medians are about $1/3$ and the worst case has a probability of about $1/8$. This substantiates our intuition that instances with unique satisfying assignments are generally the most difficult for the quantum adiabatic algorithm.

At the running times explored in this paper transitions out of the instantaneous ground state are not uncommon. In the case of a unique satisfying assignment such a transition (assuming no transition back) leads to a final state that does not correspond to the satisfying assignment. In the case of multiple satisfying assignments such transitions may lead to states that are headed towards the subspace spanned by the satisfying assignments. This is why the success probabilities are typically higher when there is more than one satisfying assignment.

9 Discussion

We have presented numerical evidence that a quantum adiabatic evolution algorithm can solve instances of satisfiability in a time that grows slowly as a function of the number of bits. Here we have worked out to 16 bits, but with more computing power instances with higher numbers of bits can be studied.

This algorithm operates in continuous time. The algorithm can be written as a product of

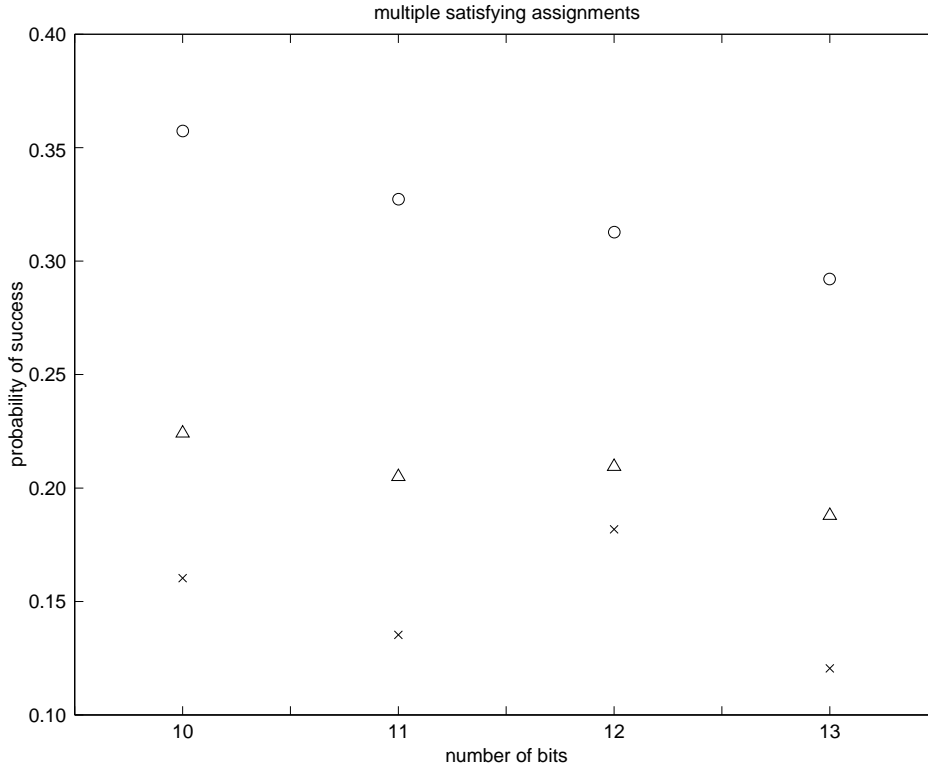


Figure 8: Exact Cover with three-bit clauses and 6, 7, 8, or 9 satisfying assignments. Circles represent the medians of 100 instances at each number of bits. Triangles are the 10th lowest probabilities and X's are the lowest. Compare with Figure 4.

few-qubit unitary operators [1] where the number of factors in the product is of order $T(n)^2$ times a polynomial in n . However, understanding the idea behind the algorithm is obscure in the conventional quantum computing paradigm. (A quantum algorithm for satisfiability that is explicitly within the ordinary paradigm is presented in [4].)

The algorithm studied in this paper works by having the quantum system stay close to the ground state of the time-dependent Hamiltonian that governs the evolution of the system. We imagine that protecting a device that remains in its ground state from decohering effects may be easier than protecting a device that requires the manipulation of excited states.

Acknowledgments

This work was supported in part by the Department of Energy under cooperative agreement DE-FC02-94ER40818. E.F. thanks the participants at the Aspen Center for Physics meeting on Quantum Information and Computation (June 2000) for many helpful discussions. We thank Mehran Kardar, Joshua Lapan, Seth Lloyd, Andrew Lundgren, and Daniel Preda for valuable input.

References

- [1] E. Farhi, J. Goldstone, S. Gutmann, and M. Sipser, “Quantum Computation by Adiabatic Evolution”, quant-ph/0001106.
- [2] C.H. Bennett, E. Bernstein, G. Brassard, and U.V. Vazirani, “Strengths and Weaknesses of Quantum Computing”, quant-ph/9701001, SIAM J. Comput. **26**, 1510 (1997).
- [3] E. Farhi and S. Gutmann, “An Analog Analogue of a Digital Quantum Computation”, quant-ph/9612026; Phys. Rev. A **57**, 2403 (1998).
- [4] T. Hogg, “Quantum search heuristics”, Phys. Rev. A **61**, 052311 (2000); T. Hogg and D. Portnov, “Quantum Optimization”, quant-ph/0006090.